Reliability&Risk Management

ANALYSIS OF THE PAPER: PROBABILISTIC ESTIMATION OF REMAINING LIFE OF A PIPELINE IN THE PRECENCE OF ACTIVE CORROSION DEFECTS

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1. **Summary**

The present work is basically focused on the analysis of a technical paper developed by M. Ahammed, from the University of Newcastle – Australia, which shows *a probabilistic approach for the assessment of remaining life of a pressurized pipeline containing active corrosion defects*.

The methodology proposed by M. Ahammed arises from a failure pressure model, based on fracture mechanics, in which the remaining strength of a corroded pipeline depends on several variables. In this case, contrary to the traditional method of estimation of remaining life in corroded pipelines (ASME B31G), these variables are recognized as random variables and so is the remaining strength.

Because of this important consideration, the methodology considers the use of statistical techniques to evaluate uncertainty propagation in the estimation of the present remaining strength and reliability analysis to predict its future behavior. Such a prediction becomes a very useful tool from the pipeline operator's perspective because it allows him to estimate safe operating pressures at any time and to prepare effective and economic inspection, repair and replacement operation schedules.

The Ahammed's work will be analyzed from two different perspectives:

- Analysis of the physical model (section three (3))
- Analysis of the statistical techniques used to evaluate the uncertainty propagation and reliability analysis. (sections four (4) and five (5))

Additionally, in section six, some improvements concerning the physical model and also two different methods for the statistical analysis are proposed. These improvements are shown using the data provided by Ahammed and also using data coming from the oil industry, specifically, from a high-pressure transmission gas pipeline.

2. **Background of the work**

2.1. Motivation: It is a well-known fact that steel pipelines are very important structures in the process industry and that a very important part of the total profit of these industries depends on the reliability of these pipelines. It is also known that as all other structures, pipelines deteriorate over the time and in this deterioration process usually the dominant mechanism is *corrosion*. Therefore, the operation and maintenance of pressured pipelines is a major concern and it becomes a risky task in aged pipelines because of the corrosion and its potential damaging effects.

All the facts previously mentioned support the need of a methodology to evaluate the pipeline current reliability and also its time dependent change.

2.2. Previous works: Among the available techniques to obtain an estimate of the remaining strength of a pipeline containing corrosion defects, the most widely accepted and used is the ANSI/ASME B31G (Manual for Determining the Remaining Strength of Corroded Pipelines, 1984), (

see apendix # 2), which is resumed in the flow diagram shown in fig. 1

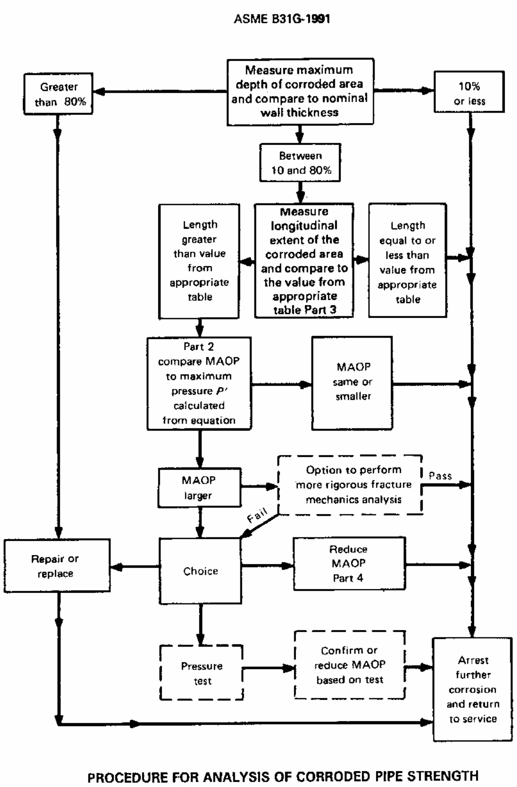


fig.1

The B31G as most of the available methods is a *deterministic approach* which uses nominal values for the parameters considered in strength and stress calculation.

3. The proposed Physical Model

3.1. Failure Pressure Model

The failure pressure model presented by M. Ahammed is based on the fundamental mechanical theory of "thin-walled cyilinders". Cylinders having internal-diameter-to-thickness ratios (D/T) grater than 10 are usually considered thin-walled cylinders. Pipelines are usually treated as such.

The most important consideration about thin-walled cylinders is that the assumption of constant stress across the wall results in a negligible error.

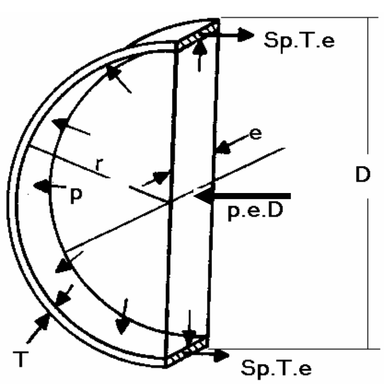


fig.3

The fig.3 shows the acting forces and stresses over a half of a thin-walled cylinder, and its dimensions:

Sp = circumferential or hoop stress

p = internal pressure

D = pipeline internal diameter

e = unit length

T = wall thickness

The equilibrium equation reveals that: Sp.(T.e) + Sp.(T.e) = p.(D.e) ; then

$$Sp := \frac{p \cdot D}{2 \cdot T} \quad \textbf{(i)}$$

Making $Sp_{stress} = Sp_{strength}$, we get:

$$pf := \frac{2 \cdot Sp_{strength}}{D} \quad \textbf{(ii)}$$

where, **pf = failure pressure**

Equation (i) and (ii) are based on the following considerations:

- External pressure is negligible compared to internal pressure
- The material obeys Hooke's law.
- The radial stress in negligible
- The normal or circumferential stress is linear
- Fluid density is relatively small compared o fluid pressure
- The shell is assumed perfectly round and of uniform thickness.

Among all the assumptions previously considered, the most critical is the last one, which assumes the shell of the pipeline as "perfectly round and of uniform thickness". This assumption is not longer valid since it is evident that volume defects are generated by corrosion process, affecting both; the roundness and the wall thickness of the pipeline.

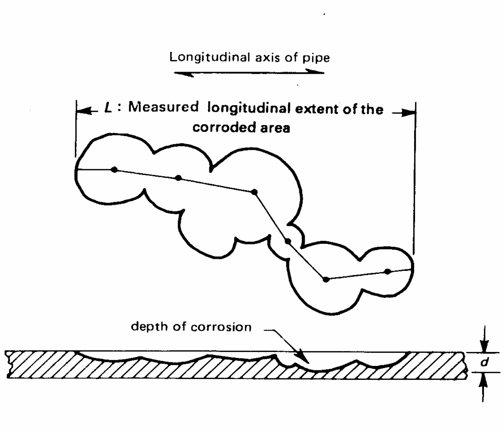


fig.4

Fig.4 shows one of such defects, where

- T=wall thickness
- L=defect length
- d=defect depth

Due to the precence of a defect, normal hoop force trajectories along the length and the depth of the defect are interrupted. These interrupted hoop force trajectories are then redistributed around the defect. This produces a region of high stress concentration, which eventually may lead failure of the pipeline if the magnitude of the concentrated stress at any region around the defect becomes higher than the pipeline strength ($Sp_{strength}$).

From the above consideration, it is clear that the pipeline strength “$Sp_{strength}$“ is affected by the presence of corrosion defects. Regarding this particular issue, several researchers have proposed different equations to predict the $Sp_{strength}$ of a pipeline containing a finite longitudinal corrosion defect. Among these models the most widely used was developed by Mok (reference1)

$$Sp_{strength} := Sf \cdot \left[\frac{1 - \left(\frac{A}{Ao}\right)}{1 - \frac{A}{(Ao \cdot M)}} \right] \qquad \textbf{(iii)}$$

where:

- $Sp_{strength}$ = Predicted hoop stress level at failure (Mpa)
- Sf = flow strength of the pipe material
- A = Projected area of defect on an axial plane through the wall thickness (mm^2)
- Ao = Original cross-sectional area of the pipe at the defect (mm^2)
- M = Folias or bulging factor

About equation (iii) there are other considerations.

- *Consideration #1*

Areas “A” and “Ao” can be obtained by the following relationships:

Ao = L . T **(iv)** (see fig. 4)

A = L . d **(v)** (see fig. 4)

Equation (v) is suggested in B31G Method, (see apendix #1) to calculate the metal area loss from the overall axial length (L) and the average depth (d) of the defect.

Fortunately, corrosion defects in pipelines can be detected by the use of a high-resolution magnetic “pic, which can be used to locate and to measure the size of a corrosion defect. Through periodic inspections, the growth of a corrosion defect can also be monitored using ultrasonic techniques. Substituing equations (iv) and (v) into equation (iii), it becomes:

$$Sp_{strength} := Sf \cdot \left[\frac{1 - \left(\frac{d}{T}\right)}{1 - \frac{d}{(T \cdot M)}} \right] \qquad \textbf{(vi)}$$

- *Consideration #2*

The flow strength (Sf) is a material property and is related to material yield strength (Sf). There are several relatipnships avalable in the literature but the most recent researches about this topic (references 2,3,4 and 5) propose the following experimentally obtained expression:

Sf = Sy + 68.95 Mpa **(vii)** ; where Sy = Yield Strength of the pipeline material

This relationship is particularly appropriated for the carbon steels commonly used in pipelines

By substitution of equation (vii) into equation (vi), we get the following expression:

$$Sp_{strength} := (Sy + 68.95)\cdot\left[\frac{1-\left(\frac{d}{T}\right)}{1-\frac{d}{(T\cdot M)}}\right] \qquad \textbf{(viii)}$$

- *Consideration #3*

The folias factor (M) is the value, which account for the stress concentration around the defect. There have been developed several expressions for the folias factor, but the last, more exact and less conservative approximation of "M" was proposed by Kiefner and Vieth (references 7 and 8), and it is expressed as follow:

$$M := \sqrt{\left[1+0.6275\cdot\frac{L^2}{D\cdot T}-0.003375\left(\frac{L^4}{D^2\cdot T^2}\right)\right]} \qquad \textbf{(ix)}$$

where:

- T=wall thickness
- L=defect length
- D=pipe diameter

By substitution of equation (ix) into equation (viii), we get the following expression:

$$Sp_{strength} := (Sy + 68.95)\cdot\left[\frac{1-\left(\frac{d}{T}\right)}{1-\frac{d}{\left[T\cdot\sqrt{\left[1+0.6275\cdot\frac{L^2}{D\cdot T}-0.003375\left(\frac{L^4}{D^2\cdot T^2}\right)\right]}\right]}}\right] \qquad \textbf{(x)}$$

Equation (x) express the pipeline strength ($Sp_{strength}$) taking into account the stress concentration around the corrosion defects, therefore it can be substituited into equation (ii) to form *the failure pressure model* presented by M. Ahammed, which can be express as follow:

$$pf := 2\cdot(Sy + 68.95)\cdot\left(\frac{T}{D}\right)\cdot\left[\frac{1-\left(\frac{d}{T}\right)}{1-\frac{d}{\left[T\cdot\sqrt{\left[1+0.6275\cdot\frac{L^2}{D\cdot T}-0.003375\left(\frac{L^4}{D^2\cdot T^2}\right)\right]}\right]}}\right] \qquad \textbf{(xi)}$$

3.2. Corrosion Model

Corrosion is the destructive attack of a metal by chemical or electrochemical reaction with its environment. The major classifications of corrosion given by Fontana (Reference # 9) are:

- Erosion corrosion
- Galvanic corrosion
- Uniform attack
- Pitting corrosion
- Cavitation
- Crevice corrosion
- Stress corrosion cracking
- Selective leaching
- Intergranular corrosion

Corrosion is such a complex failure mechanism, which can damage a metal surface in many different ways. The work developed by M. Ahammed is particularly concern with those types of corrosion which produce macroscopic defects due to material losses, because these defects results in the reduction of the metal cross section with its correspondent reduction of the pipeline strength ($Sp_{strength}$. Such corrosion types are erosion, pitting corrosion and uniform attack.

This approach is not because of simplicity, but because of the experience in analyzing pipeline failures, which has shown that for most of the common industrial applications, pipeline failures aren't due to other types of corrosion than the ones mentioned. Nevertheless, for complex applications, as nuclear applications or operating conditions with extremely high stress solicitude, a more extensive analysis is required.

The Ahammed's considerations about corrosion are focused in the growth rate of macro-defects.

The growth of macro defects is directly related with the exposure period and depends on the characteristics of the pipeline material, properties of the fluid being transported and the surrounding environment. It has been found that this rate is high during an initial period and then gradually decrease to finally reach a steady-state rate.

The researches cited by Ahammed (Southwell et al. Reference # 10) shown that the initial period of relatively high corrosion rate, (about a year in average), is not of much concern because during this period the defects are usually small and hence do not pose much threat to pipeline integrity.

As the exposure period increases, the growth rate decreases, but the overall size of corrosion defect increases and becomes a greater risk for the pipeline integrity. By this time, the steady-state growth rate is a good approximation.

The following expressions are suggested for the growth rate during the steady state period:

$$Rd := \frac{d - do}{t - to} \qquad \text{then,} \qquad d := do + Rd \cdot (t - to) \qquad \textbf{(xii)}$$

Where:

- Rd=steady state corrosion rate in the direction of depth
- do=measured depth of the defect at the time t=to

$$Rl := \frac{L - Lo}{t - to} \qquad \text{then,} \qquad L := Lo + Rl \cdot (t - to) \qquad \textbf{(xiii)}$$

Where:

- Rl=steady state corrosion rate in the direction of length
- Lo=measured length of the defect at the time t=to

Other approach for the estimation of the corrosion growth rate will be proposed in Appendix #2

.

3.3. Failure pressure model + corrosion defect growth model:

Finally, by substituing equations (xii) and (xiii) in equation (xi), we get the failure pressure model which take into account the effect of continued corrosion.

$$pf := 2 \cdot (Sy + 68.95) \cdot \left(\frac{T}{D}\right) \cdot \left[\frac{1 - \left[\frac{(do + Rd \cdot t)}{T}\right]}{1 - \frac{(do + Rd \cdot t)}{\left[T \cdot \sqrt{\left[1 + 0.6275 \frac{(Lo + Rl \cdot t)^2}{D \cdot T} - 0.003375 \left[\frac{(Lo + Rl \cdot t)^4}{D^2 \cdot T^2}\right]\right]}\right]}}\right] \qquad \textbf{(xiv)}$$

4. **Statistical and Reliability Analysis**

4.1. Introduction

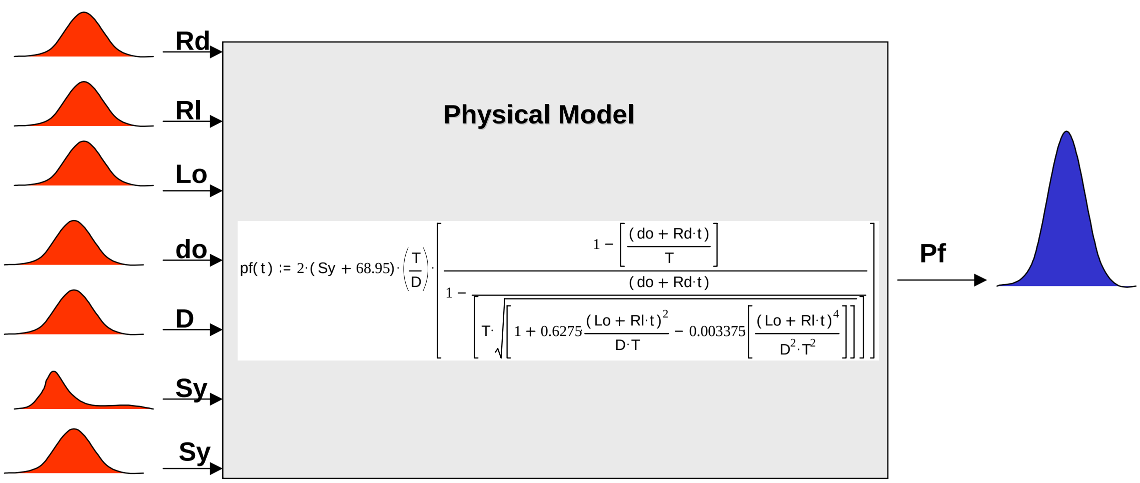
In a traditional approach, the variables of equation (xiv) are treated as deterministic values; therefore, the obtained failure pressure is also a deterministic value. However, in reality these parameters show certain degree of variability in their values.

The M. Ahammed's approach recognizes such values as random variables, (each one with a probability density function, a mean and a variance), therefore the resulting failure pressure is a random variable with its own degree of variability. This variability is the most important factor when estimating the remaining life of a pipeline, and the estimation method of such variability is the most valuable contribution of the proposed approach.

4.2. Methodology

4.2.1 Step #1: Uncertainty Propagation

Fig. 5 shows that if the physical model's inputs are random variables, then, its output must be a random variable too. The methods to determine the random characteristics of the outpus given the random characteristics of the imputs are known as "uncertainty propagation methods" (u.p.m).



The u.p.m selected by M. Ahammed is know as "advanced first order second moment method". This method is based on the Taylor series:

$$f(x_1, x_2, \dots, x_n) = f(\mu_1, \mu_2, \dots, \mu_n) + (x_1 - \mu_1)\,\frac{\partial f}{\partial x_1}\Big|_{\mu_1,\mu_2,\dots,\mu_n}, + \cdots + (x_n - \mu_n) \qquad \textbf{(xv)}$$

Where X_1, X_2, X_3,...., X_n are the "n" random variables or imputs and μ_1, μ_2, μ_3,...., μ_n are their respective mean values.

If the higher terms are neglected and the variables are assumed to be statistically independent, the mean and the variance of the function are given by:

$$\mu_f = f(\mu_1, \mu_2, \dots, \mu_n) \qquad \textbf{(xvi)}$$

$$\sigma_f^2 = \sigma_{x1}^2 \left(\frac{\partial f}{\partial x_1}\Big|_{\mu_1,\mu_2,\dots,\mu_n} \right)^2 + \cdots + \sigma_{xn}^2 \left(\frac{\partial f}{\partial x_n}\Big|_{\mu_1,\mu_2,\dots,\mu_n} \right)^2 \qquad \textbf{(xvii)}$$

A continuation, this method is run for the data provided by M Ahammed and the physical model of equation (xiv). It is done using MathCad 5.

Tabla 1 shows the values and characteristics of the inputs random variables provided by M Ahammed.

Variable	Mean Value	Standar Deviation	pdf
Sy	423	0.002239477	Log - Normal
Rd	0.1	0.02	Normal
Rl	0.1	0.02	Normal
do	3	0.3	Normal
Lo	200	10	Normal
D	600	18	Normal
T	10	0.5	Normal

table 1

4.2.1.1. Example 1: Estimation of μ_{Pf} and σ_{Pf} for t = 30 years

Approximate Method (Taylor Serie)

$t := 30$

$Do := 3$ $S_4 := 0.3$ $Rd := 0.1$ $S_2 := 0.02$ $T := 10$ $S_1 := 0.5$

$D := 600$ $S_5 := 18$ $Rl := 0.1$ $S_3 := 0.02$

$Lo := 200$ $S_7 := 10$ $Sy := 423$ $S_6 := 0.002239477497$

$Pa := 5$

$$Pf := \left[2\cdot(Sy+68.95)\cdot\frac{T}{D}\right]\cdot\frac{1-\frac{(Do+Rd\cdot t)}{T}}{1-\frac{(Do+Rd\cdot t)}{T\cdot\left[\sqrt{\left[1+0.6275\cdot\frac{((Lo+Rl\cdot t))^2}{D\cdot T}-0.003375\cdot\left[\frac{((Lo+Rl\cdot t))^4}{D^2\cdot T^2}\right]\right]}\right]}} \qquad Pf = 8.917$$

$$C_1 := \frac{d}{dT}\left[\left[2\cdot(Sy+68.95)\cdot\frac{T}{D}\right]\cdot\frac{1-\frac{(Do+Rd\cdot t)}{T}}{1-\frac{(Do+Rd\cdot t)}{T\cdot\left[\sqrt{\left[1+0.6275\cdot\frac{((Lo+Rl\cdot t))^2}{D\cdot T}-0.003375\cdot\left[\frac{((Lo+Rl\cdot t))^4}{D^2\cdot T^2}\right]\right]}\right]}}\right] \qquad C_1 = 2.033$$

$$C_2 := \frac{d}{dRd}\left[\left[2\cdot(Sy+68.95)\cdot\frac{T}{D}\right]\cdot\frac{1-\frac{(Do+Rd\cdot t)}{T}}{1-\frac{(Do+Rd\cdot t)}{T\cdot\left[\sqrt{\left[1+0.6275\cdot\frac{((Lo+Rl\cdot t))^2}{D\cdot T}-0.003375\cdot\left[\frac{((Lo+Rl\cdot t))^4}{D^2\cdot T^2}\right]\right]}\right]}}\right] \qquad C_2 = -50.852$$

$$C_3 := \frac{d}{dRl}\left[\left[2\cdot(Sy+68.95)\cdot\frac{T}{D}\right]\cdot\frac{1-\frac{(Do+Rd\cdot t)}{T}}{1-\frac{(Do+Rd\cdot t)}{T\cdot\left[\sqrt{\left[1+0.6275\cdot\frac{((Lo+Rl\cdot t))^2}{D\cdot T}-0.003375\cdot\left[\frac{((Lo+Rl\cdot t))^4}{D^2\cdot T^2}\right]\right]}\right]}}\right] \qquad C_3 = -0.367$$

$$C_4 := \frac{d}{dDo}\left[\left[2\cdot(Sy+68.95)\cdot\frac{T}{D}\right]\cdot\frac{1-\frac{(Do+Rd\cdot t)}{T}}{1-\frac{(Do+Rd\cdot t)}{T\cdot\left[\sqrt{\left[1+0.6275\cdot\frac{((Lo+Rl\cdot t))^2}{D\cdot T}-0.003375\cdot\left[\frac{((Lo+Rl\cdot t))^4}{D^2\cdot T^2}\right]\right]}\right]}}\right] \qquad C_4 = -1.695$$

$$C_5 := \frac{d}{dD}\left[\left[2\cdot(Sy+68.95)\cdot\frac{T}{D}\right]\cdot\frac{1-\frac{(Do+Rd\cdot t)}{T}}{1-\frac{(Do+Rd\cdot t)}{T\cdot\left[\sqrt{\left[1+0.6275\cdot\frac{((Lo+Rl\cdot t))^2}{D\cdot T}-0.003375\cdot\left[\frac{((Lo+Rl\cdot t))^4}{D^2\cdot T^2}\right]\right]}\right]}}\right] \qquad C_5 = -0.013$$

$$C_6 := \frac{d}{dSy}\left[\left[2\cdot(Sy+68.95)\cdot\frac{T}{D}\right]\cdot\frac{1-\frac{(Do+Rd\cdot t)}{T}}{1-\frac{(Do+Rd\cdot t)}{T\cdot\left[\sqrt{\left[1+0.6275\cdot\frac{((Lo+Rl\cdot t))^2}{D\cdot T}-0.003375\cdot\left[\frac{((Lo+Rl\cdot t))^4}{D^2\cdot T^2}\right]\right]}\right]}}\right] \qquad C_6 = 0.018$$

$$C_7 := \frac{d}{dLo}\left[\left[2\cdot(Sy+68.95)\cdot\frac{T}{D}\right]\cdot\frac{1-\frac{(Do+Rd\cdot t)}{T}}{1-\frac{(Do+Rd\cdot t)}{T\cdot\left[\sqrt{\left[1+0.6275\cdot\frac{((Lo+Rl\cdot t))^2}{D\cdot T}-0.003375\cdot\left[\frac{((Lo+Rl\cdot t))^4}{D^2\cdot T^2}\right]\right]}\right]}}\right] \qquad C_7 = -0.012$$

$$Var := \left[\sum_{i=1}^{7}\left[(C_i)^2\cdot(S_i)^2\right]\right]$$

$\mu pf := Pf$ $\qquad \mu pf = 8.917$

$\sigma pf := \sqrt{Var}$ $\qquad \sigma pf = 1.547$

This method can be applied for any value of the time "t" in order to estimate the mean and the variance of the failure pressure or remaining strength. Table 2 shows the results gotten for t = 20,30,40 and 50 years

Time "t" (years)	$\mu_{Pf\ (Mpa)}$	$\sigma_{Pf\ (Mpa)}$	cov=σ_{Pf}/μ_{Pf}
20	10.527	1.259	0.1196
30	8.917	1.547	0.1734
40	7.102	1.978	0.2785
50	5.044	2.57	0.5095

table 2

From table 2 it can be noticed that there is a clear "drift" in the mean of the remaining strength and also the dispersion of the its distribution increases with time (diffusion). Both trends are shown in fig.6

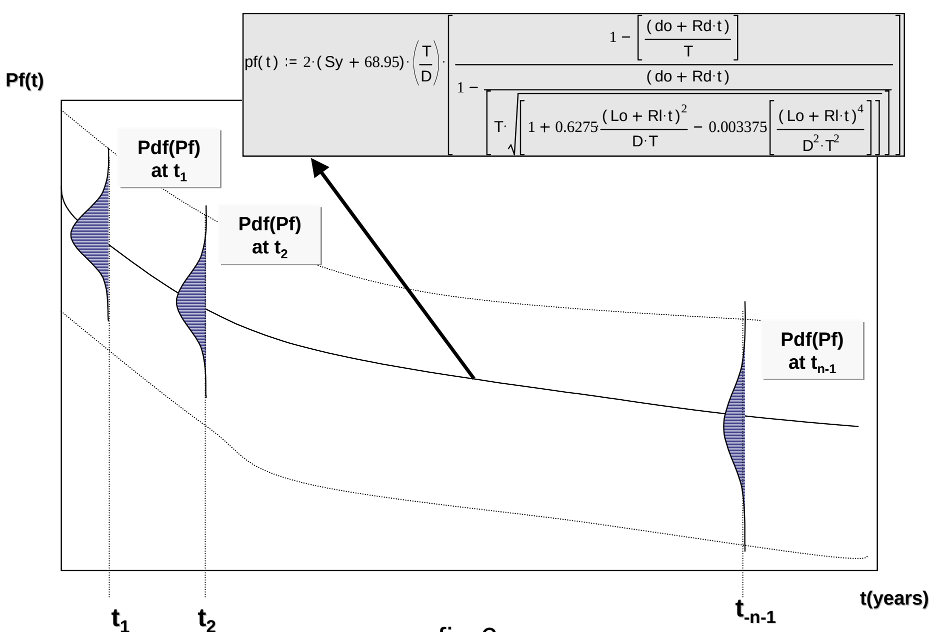


fig.6

In Fig. 6 it is assumed a Normal or Gaussian as the distribution of "Pf".

At this point it is important to say that the method proposed by M. Ahammed is able to give us information about the values of the mean and the standard deviation of the failure pressure at any time, but doesn't give information about the probability distribution model that better fits the distribution of "Pf". Therefore the confident level of this estimation can not be calculated precisely. In addition the coefficient of variation (cov), shown in table 2, can be interpreted as a measure of the confidence of our estimation.

In section six (6) a Montecarlo approach is proposed to avoid the limitations mentioned.

4.2.2. Step #2: Stress-Strength Interference Reliability Analysis

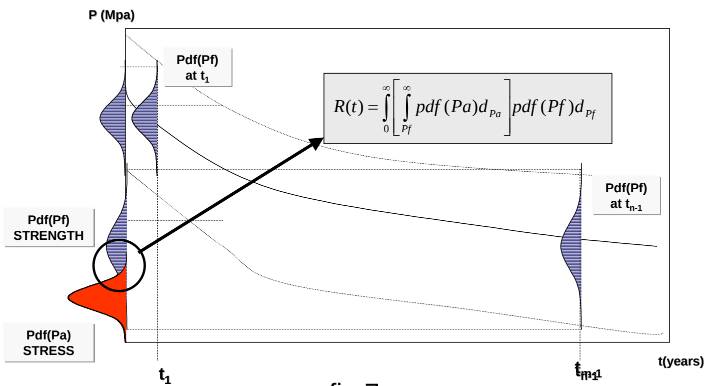


fig.7

Fig. 7 shows that the mean value of the remaining strength "Pf" decreases with the time increment , and also increases its dipersion. At time t = t_1 , the distribution of "Pf" (pdf(Pf)) is still far from the stress distribution(pdf(Pa)), (Pa = service pressure), but at time t = t_n –1 there are certain degree of overlap between both distributions. This degree of overlap is proportional to the probability of failure

The estimation of the reliability of the pipeline in presence of active corrosion defects can be obtained by the expression show in fig. 7

$$R(t)=\int_0^{\infty}\left[\int_{Pf}^{\infty} pdf\ (Pa)\,d_{Pa}\right]pdf\ (Pf)\,d_{Pf} \qquad \textbf{(xviii)}$$

To solve the integral of equation (xviii) it is necessary to know the pdf of Pf, which is not possible with the method, described in step 1; nevertheless, in the Ahammed approach an approximation is proposed by defining the random variable "Z"

Z = Pf – Pa **(xix)**

Assuming Z to be normally distributed, the pipeline failure probability corresponding to the defect "i" is defined as

$$P_i = \text{Prob. } (Z<0) = \phi\left[\frac{(Pf - Pa)}{\sqrt{\sigma pf^2 + \sigma pa^2}}\right] \qquad \textbf{(xx)}$$

$$\text{Then, } R_i = 1 - \phi\left[\frac{(Pf - Pa)}{\sqrt{\sigma pf^2 + \sigma pa^2}}\right] \qquad \textbf{(xxi)}$$

The continuation of example 1, for t=20 and Pa = N (5; 0.5) (Normal distribution with mean = 5 MPa and standard deviation = 0.5 Mpa), shows the following results:

$$\beta := \frac{(Pf - Pa)}{\sqrt{Var + 0.5^2}} \qquad \beta = 4.081$$

$$F := 1 - pnorm(\beta, 0, 1) \qquad Rel := 1 - F$$

$$F = 2.238 \cdot 10^{-5} \qquad Rel = 1$$

Table 3 shows the results gotten for t = 20,30,40 and 50 years

Time "t" (years)	β	F_i	R_i
20	4.081	$2.238.10^{-5}$	1
30	2.409	0.008	0.992
40	1.030	0.141	0.849
50	0.017	0.493	0.507

table 3

4.2.3. Step # 3: Sensitivity Analysis:

The sensitivity analysis allows us to know the relative contributions of the various individual random variables to the variance of the failure function. Each contribution can be calculated using the following expression:

$$\alpha_{X_i}^2 = \frac{\left(\frac{\partial f(x_1, x_2, x_3, ...x_n)}{\partial x_i}\right)^2}{\sigma_f^2} \qquad \textbf{(xxii)}$$

In the following page, the sensitivity analysis for the failure pressure model is shown, for a time period t=20 years.

The analysis showed that in this case only the dispersion of the values of the wall thickness "T", the radial corrosion growth rate "Rd" and the defect depth "do", have a considerable effect in the dispersion of the values of the remaining strength "Pf" and in consequence, in the reliability estimation of the pipeline.

Sensitivity Analysis

Contribution of the wall thicknes "T"

$$\alpha_1 := \sqrt{\left[\frac{(C_1)^2 \cdot (S_1)^2}{\sigma pf^2}\right]} \qquad \alpha_1 = 0.764$$

Contribution of the radial corrosion growth rate"Rd"

$$\alpha_2 := \sqrt{\left[\frac{(C_2)^2 \cdot (S_2)^2}{\sigma pf^2}\right]} \qquad \alpha_2 = 0.479$$

Contribution of the longitudinal corrosion growth rate"Rl"

$$\alpha_3 := \sqrt{\left[\frac{(C_3)^2 \cdot (S_3)^2}{\sigma pf^2}\right]} \qquad \alpha_3 = 3.639 \cdot 10^{-3}$$

Contribution of the corrosion depth at t = to "Do"

$$\alpha_4 := \sqrt{\left[\frac{(C_4)^2 \cdot (S_4)^2}{\sigma pf^2}\right]} \qquad \alpha_4 = 0.359$$

Contribution of the pipe diameter "D"

$$\alpha_5 := \sqrt{\left[\frac{(C_5)^2 \cdot (S_5)^2}{\sigma pf^2}\right]} \qquad \alpha_5 = 0.223$$

Contribution of the yield strength of the pipeline material "Sy"

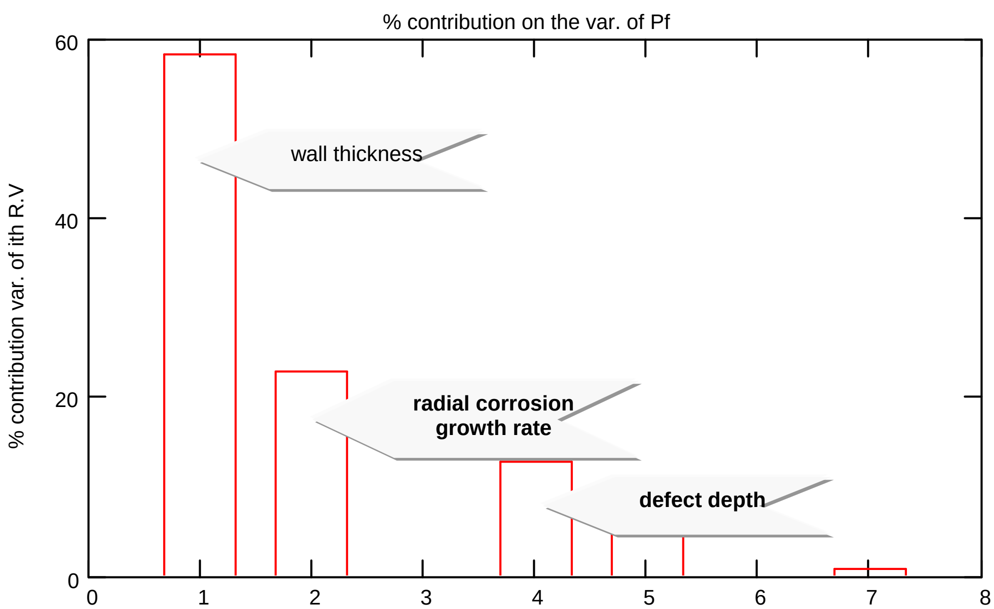
$$\alpha_6 := \sqrt{\left[\frac{(C_6)^2 \cdot (S_6)^2}{\sigma pf^2}\right]} \qquad \alpha_6 = 3.808 \cdot 10^{-5}$$

Contribution of the corrosion length at t = to "Lo"

$$\alpha_7 := \sqrt{\left[\frac{(C_7)^2 \cdot (S_7)^2}{\sigma pf^2}\right]} \qquad \alpha_7 = 0.091$$

$i := 1, 2 .. 7$

$$\left[\sum_{i=1}^{7} \left[(\alpha_i)^2\right]\right] = 1 \qquad \%_i := 100 \cdot (\alpha_i)^2$$



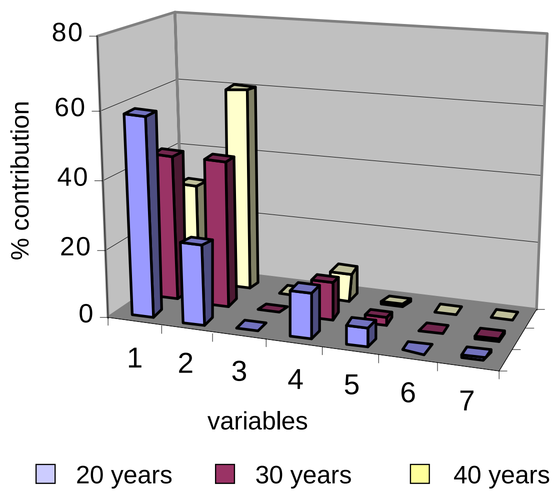


fig.8

Fig. 8 shows the contributions of the variables to the variance of the remaining strength "Pf" for different periods of time.

It is clear that the major contributions come from the wall thickness "T", the radial corrosion growth rate "Rd", and the defect depth "do", (variables 1, 2 and 4 respectively), consistently over the time

4.2.4. Step # 4: System Analysis

The system analysis perspective is required when considering more than one defect per pipeline, which is normally the case. In such case, the pipeline can be modeled as a serial system because each defect is able to cause the pipeline to fail.

Let us designate these defects by 1,2,3.......n and the corresponding failure probabilities by P_{f1},P_{f2} ,P_{f3}.............. P_{fn} respectively. Then the probability of failure for the pipeline can be estimated by:

P_f(pipeline) = 1-(1- P_{f1}).(1- P_{f2}).(1- P_{f3})................(1- P_{fn}) **(xxiii)**

5**. Limitations of the proposed approach**

At this point it is important to summarize the limitations of the approach proposed by M. Ahammed in order to establish the boundaries for its reasonable use.

5.1. Limitations about the proposed Physical Model

- The physical model proposed accounts only for those types of corrosion, which produce macroscopic defects due to material losses, such as corrosion erosion, pitting corrosion and uniform attack. For complex applications such as nuclear plants or operating conditions with extremely high strength solicitude, other corrosion mechanisms such as intergranular corrosion, stress corrosion cracking or selective leaching must be considered; then, a more extensive analysis is required.

5.2. Limitations about the proposed method for statistical and reliability analysis

- The " Advanced first order-second moment method" proposed to estimate the resulting variability of the remaining strength (Pf) given the variability of the input values, allows for the estimation of its mean value and its standard deviation, but is not able to provide its "distribution function" or pdf (Pf). In consequence, a precise calculation of the confidence interval for this estimate is not possible. In section six (6) "Proposed Improvements", an approach to avoid this limitation is proposed.

6. Proposed Improvements

6.1. About the Physical Model

6.1. 1. Incorporation of an Erosion Corrosion Physics Based Model:

The availability of relationships defining either mass transfer or erosion corrosion fron surfaces is extremaly limited, even though, recent work has indicated that once a surface is roughened, the rate of mass transfer is governed by roughness, not the geometry of the surface. More importantly, the results indicate that a universal relationship for the mass trensfer for roughened surfaces may exist. The reaction rate is governed by the Sherwood number (Sh), given as:

$$Sh = \frac{K.d}{D}$$ (xxiv) , where:

K= mass transfer coefficient

d=characteristic speciment length dimension

D=diffusivity of the relevant species

$Sh = C.\mathrm{Re}^{x}.Sc^{y} =$ Sherwood number (xxv)

Re= Reynolds number

$Sc = \frac{\gamma}{D} =$ Schmidt number

$\gamma =$ kinematic viscosity

6.1. 2. Approach for the statistical estimation of wall thickness loss due to corrosion

The linear model assumed to predict the corrosion growth rate is currently under discussion. In appendix #1 an extensive analysis of data coming from different services in the oil industry, shows that a more accurate exponential model can be used.

6.2. About the Statistical and Reliability Analysis

As it was mentioned in section five (5), the "advanced first order-second moment method" allows us to estimate its mean value and its standard deviation of the remaining strength (Pf), but not its "distribution function" or pdf (Pf). Therefore the level of confidence of this estimate can not be obtained.

To solve this problem, a Montecarlo Approach is proposed. This approach can be really useful if a precise estimation of the confidence interval is required. These calculation where developed using "MathCad 5".

Probabilistic estimation of remaining life of a pipeline in the precence of active corrosion defects

Terms definitions:

Sy = yield strength of pipe material

T = Wall Thickness

Do = Measured depth of the defect at t=to

Lo = Measured length of the defect at t=to

D = Pipe diameter

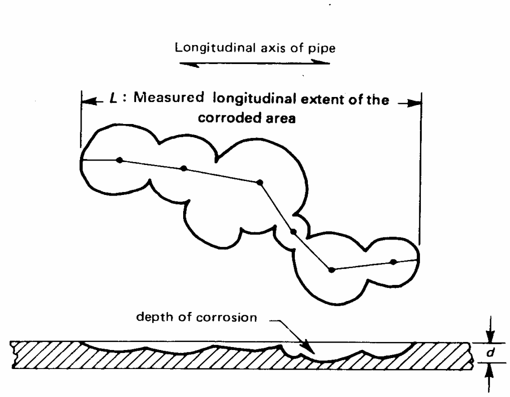
Rd = Steady-state corrosion rate in the direction of depth

Rl = Steady-state corrosion rate in the direction of length

M = Folias factor = $\sqrt{\left[1 + 0.6275\cdot\frac{L^2}{D\cdot T} - 0.003375\cdot\left(\frac{L^4}{D^2\cdot T^2}\right)\right]}$

L = length of the defect at any time (t) = Lo+Rl(t-to)

d = depth of the defect at any time (t) = Do=Rd(t-to)



1.- Montecarlo Simulation Strength (Pf)

$t := 30$ $\quad m := 100000$ $\quad Sy := rlnorm(m, \ln(422.0537611), 0.002239477497)$

$Do := rnorm(m, 3, 0.3)$ $\quad D := rnorm(m, 600, 18)$

$Lo := rnorm(m, 200, 10)$ $\quad Rd := rnorm(m, 0.1, 0.02)$ $\quad Rl := rnorm(m, 0.1, 0.02)$

$T := rnorm(m, 10, 0.5)$

$$L := \overrightarrow{(Lo + Rl \cdot t)} \qquad M := \overrightarrow{\sqrt{\left[1 + 0.6275 \cdot \frac{L^2}{D \cdot T} - 0.003375 \cdot \left(\frac{L^4}{D^2 \cdot T^2}\right)\right]}} \qquad P := \overrightarrow{\left[2 \cdot (Sy + 68.95) \cdot \frac{T}{D}\right]}$$

$$F := \overrightarrow{\frac{1 - \frac{(Do - Rd \cdot t)}{T}}{1 - \frac{(Do - Rd \cdot t)}{T \cdot M}}} \qquad Pf := \overrightarrow{\left[\left[2 \cdot (Sy + 68.95) \cdot \frac{T}{D}\right] \cdot \frac{1 - \frac{(Do + Rd \cdot t)}{T}}{1 - \frac{(Do + Rd \cdot t)}{T \cdot M}}\right]}$$

Pf =

	0
0	7.311
1	8.149
2	8.709
3	7.894
4	7.879
5	6.432
6	9.416
7	9.042
8	11.187
9	7.671
10	9.504
11	7.435
12	8.658
13	11.846

$$\mu := \left(\frac{1}{m - 1}\right) \cdot \sum_{i = 0}^{m - 1} Pf_i \qquad \mu = 8.862$$

$$\sigma := \sqrt{\left(\frac{1}{m - 2}\right) \cdot \sum_{i = 0}^{m - 1} \left(Pf_i - \mu\right)^2} \qquad \sigma = 1.551$$

1.1.- Determination of the pdf of "Pf"

Enter desired number of bins in the interval: $bin := 150$

Size, mean, and standard deviation of the data:

Size $n := length(Pf)$ $n = 1 \cdot 10^5$

Mean $\mu := mean(Pf)$ $\mu = 8.862$

SD $\sigma := stdev(Pf) \cdot \sqrt{\frac{n}{n-1}}$ $\sigma = 1.551$

Frequency distribution:

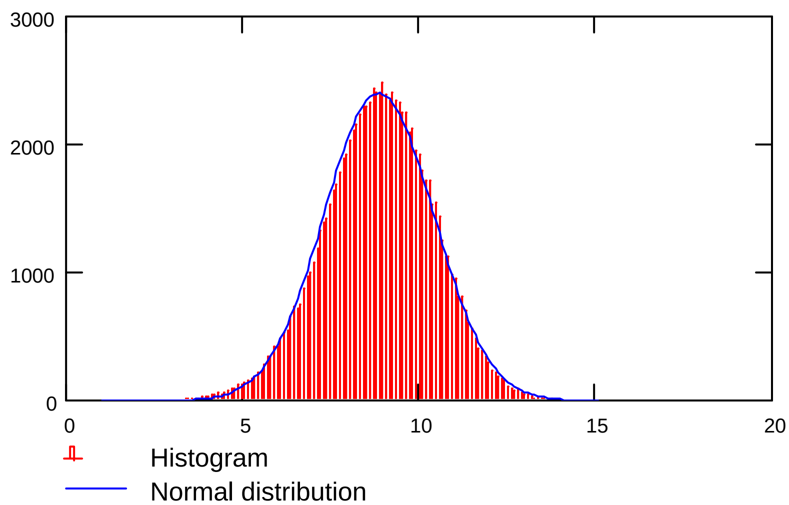
$lower := floor(min(Pf))$ $upper := ceil(max(Pf))$

$h := \frac{upper - lower}{bin}$ $j := 0..bin$

$int_j := lower + h \cdot j$

$f := hist(int, Pf)$ $int := int + 0.5 \cdot h$

$F(y) := n \cdot h \cdot dnorm(y, \mu, \sigma)$



As it can be noticed, the histogrem of the values of "Pf", makes a very good fit with the norma distribution

2.- Simulation Stress (Pa)

$Pa := rnorm(m, 5, 0.5)$

Enter desired number of bins in the interval: $bin := 150$

Size, mean, and standard deviation of the data:

Size $n := length(Pa)$ $n = 1 \cdot 10^{5}$

Mean $\mu pa := mean(Pa)$ $\mu pa = 4.999$

SD $\sigma pa := stdev(Pa) \cdot \sqrt{\frac{n}{n-1}}$ $\sigma pa = 0.502$

Pa =

	0
0	4.794
1	5.133
2	5.608
3	5.374
4	5.138
5	5.265
6	5.028
7	4.48
8	4.665
9	4.81
10	5.509
11	5.081
12	4.936
13	5.288

Frequency distribution:

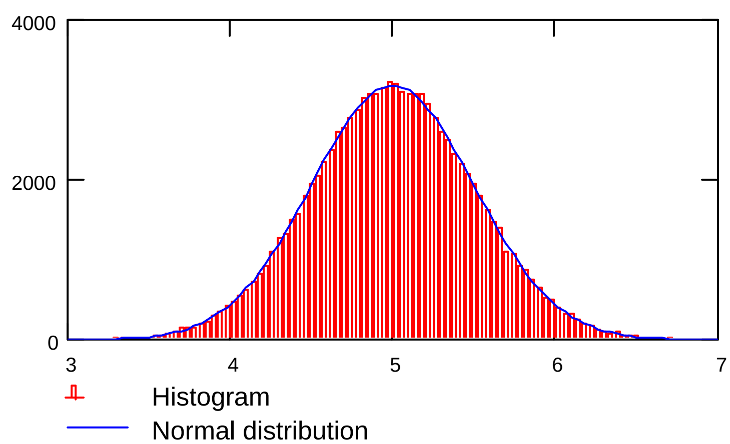
$lower := floor(min(Pa))$ $upper := ceil(max(Pa))$

$h := \frac{upper - lower}{bin}$ $j := 0 .. bin$

$int_j := lower + h \cdot j$

$f := hist(int, Pa)$ $int := int + 0.5 \cdot h$

$G(I) := n \cdot h \cdot dnorm(I, \mu pa, \sigma pa)$



3.- Stress-Strength Interference Reliability Estimation

Strength

$\sigma = 1.551 \qquad \mu = 8.862$

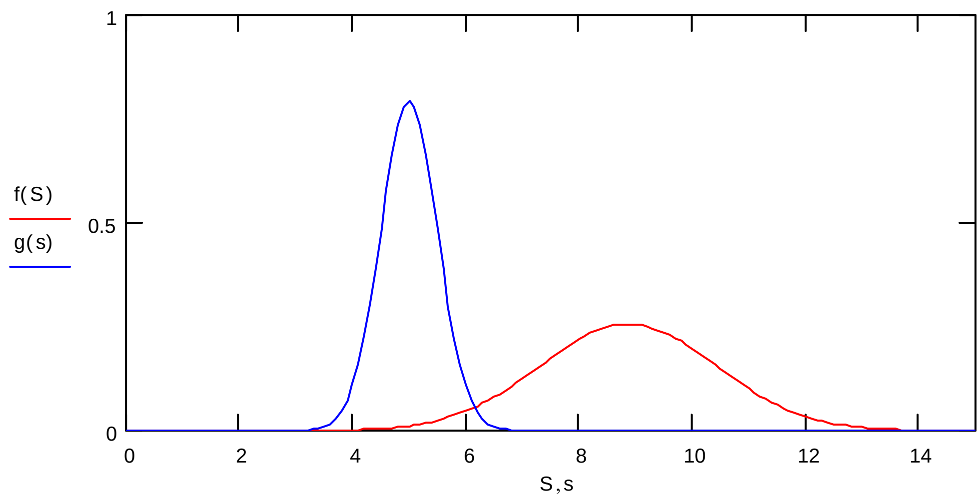
$$f(S) := \frac{1}{\sqrt{2\cdot\pi}\cdot\sigma}\cdot e^{\frac{-1}{2}\cdot\left(\frac{S-\mu}{\sigma}\right)^2} \qquad \int_{-100}^{100} f(S)\,dS = 1$$

Stress

$\sigma pa = 0.502 \qquad \mu pa = 4.999$

$$g(s) := \frac{1}{\sqrt{2\cdot\pi}\cdot\sigma pa}\cdot e^{\frac{-1}{2}\cdot\left(\frac{s-\mu pa}{\sigma pa}\right)^2} \qquad \int_{-10}^{10} g(s)\,ds = 1$$

$s := 0, 0.1 .. 20 \qquad S := 0, 0.1 .. 20$



$$Rel := \int_{-100}^{100}\left(\frac{1}{2}\cdot erf\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{\sigma pa}\cdot S - \frac{1}{2}\cdot\frac{\mu pa}{\sigma pa}\cdot\sqrt{2}\right) - \frac{1}{2}\cdot erf\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{\sigma pa}\cdot 0 - \frac{1}{2}\cdot\frac{\mu pa}{\sigma pa}\cdot\sqrt{2}\right)\right)\cdot f(S)\,dS$$

$Rel = 0.991$

4.- Determination of the failure state   Z=(Pf-Pa)

$Z := \overrightarrow{(Pf - Pa)}$

Enter desired number of bins in the interval:

bin := 150

Size $n := \text{length}(Z)$ $n = 1 \cdot 10^{5}$

Mean $\mu z := \text{mean}(Z)$ $\mu z = 3.863$

SD $\sigma z := \text{stdev}(Z) \cdot \sqrt{\frac{n}{n-1}}$ $\sigma z = 1.629$

Z =

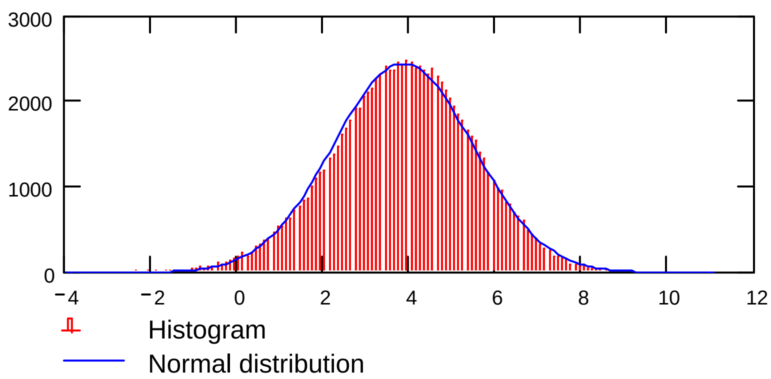
	0
0	2.517
1	3.016
2	3.1
3	2.521
4	2.741
5	1.168
6	4.388
7	4.562
8	6.522
9	2.861
10	3.995
11	2.354
12	3.723
13	6.558

Frequency distribution:

$\text{lower} := \text{floor}(\min(Z))$ $\text{upper} := \text{ceil}(\max(Z))$

$h := \frac{\text{upper} - \text{lower}}{\text{bin}}$ $j := 0 .. \text{bin}$ $\text{bin} = 150$ $\text{int}_j := \text{lower} + h \cdot j$

$f := \text{hist}(\text{int}, Z)$ $\text{int} := \text{int} + 0.5 \cdot h$ $F(x) := n \cdot h \cdot \text{dnorm}(x, \mu z, \sigma z)$



Failure Probability=F=Pr(Z>0) $F := \int_{-1000}^{0} \frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma z} \cdot e^{\frac{-1}{2} \cdot \left(\frac{Z - \mu z}{\sigma z}\right)^{2}} \, dZ$ $F = 8.859 \cdot 10^{-3}$

5.- Reliability Prediction

$m := 100000$ $\quad t1 := 1, 5 .. t + 45$ $\quad Sy := rlnorm(m, \ln(422.0537611), 0.002239477497)$

$Do := rnorm(m, 3, 0.3)$ $\quad D := rnorm(m, 600, 18)$ $\quad Rl := rnorm(m, 0.1, 0.02)$

$Lo := rnorm(m, 200, 10)$ $\quad Rd := rnorm(m, 0.1, 0.02)$ $\quad T := rnorm(m, 10, 0.5)$

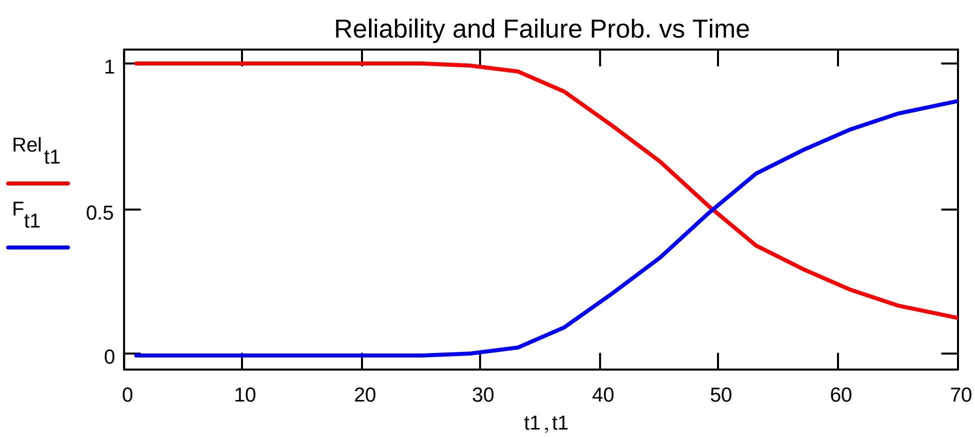
$$L_t := \overrightarrow{(Lo + Rl \cdot t1)} \qquad M_{t1} := \overrightarrow{\sqrt{\left[1 + 0.6275 \cdot \frac{(L_{t1})^2}{D \cdot T} - 0.003375 \cdot \left[\frac{(L_{t1})^4}{D^2 \cdot T^2}\right]\right]}}$$

$$Pf_{t1} := \overrightarrow{\left[\left[2 \cdot (Sy + 68.95) \cdot \frac{T}{D}\right] \cdot \frac{1 - \frac{(Do + Rd \cdot t1)}{T}}{1 - \frac{(Do + Rd \cdot t1)}{T \cdot M_{t1}}}\right]} \qquad \mu_{t1} := mean(Pf_{t1})$$

$$\sigma_{t1} := stdev(Pf_{t1}) \cdot \sqrt{\frac{n}{n-1}}$$

$$S := 0.00001, 0.1 .. 200 \qquad z(S) := \left(\frac{1}{2} \cdot erf\left(\frac{1}{2} \cdot \frac{\sqrt{2}}{\sigma pa} \cdot S - \frac{1}{2} \cdot \frac{\mu pa}{\sigma pa} \cdot \sqrt{2}\right) - \frac{1}{2} \cdot erf\left(\frac{1}{2} \cdot \frac{\sqrt{2}}{\sigma pa} \cdot 0 - \frac{1}{2} \cdot \frac{\mu pa}{\sigma pa} \cdot \sqrt{2}\right)\right)$$

$$Rel_{t1} := \int_{-0}^{100} z(S) \cdot \left[\left[\frac{1}{(\sigma_{t1} \cdot \sqrt{2 \cdot \pi})}\right] \cdot e^{\left[-\left[\frac{1}{2 \cdot (\sigma_{t1})^2}\right] \cdot \left[(S) - \mu_{t1}\right]^2\right]}\right] dS \qquad F_{t1} := 1 - Rel_{t1}$$

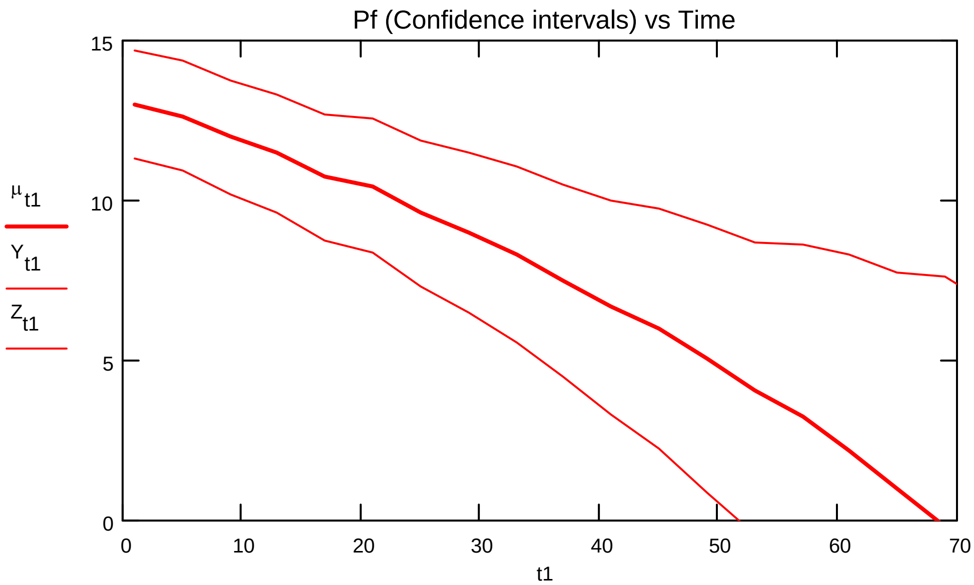


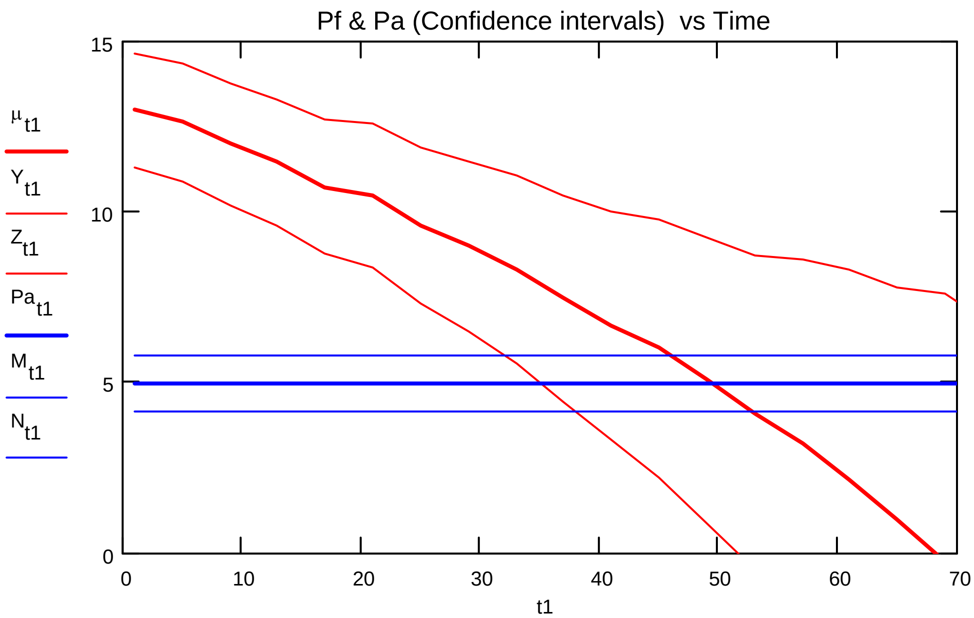
6.- Confidence Intervals Estimation

$Y_{t1} := \text{qnorm}(0.95, \mu_{t1}, \sigma_{t1})$ $Z_{t1} := \text{qnorm}(0.05, \mu_{t1}, \sigma_{t1})$

$Pa_{t1} := \mu pa$

$M_{t1} := \text{qnorm}(0.95, \mu pa, \sigma pa)$ $N_{t1} := \text{qnorm}(0.05, \mu pa, \sigma pa)$





7. Conclusions

- The use of the physical model proposed by M. Ahammed must be limited for those uses in which the dominant corrosion mechanisms are corrosion erosion, pitting corrosion and uniform attack.

  The experience has shown that these are actually the failure mechanisms for most of the pipelines industrial applications. Nevertheless, for complex applications (nuclear plants or operating conditions with extremely high strength solicitude), other corrosion mechanisms such as intergranular corrosion, stress corrosion cracking and selective leaching must be considered; then, a more extensive analysis is required.

- The proposed statistical treatment of the physical model, accounts for the uncertainty due to the variability of the input parameters, but the uncertainty associated with the model by itself is not included in this study.

  There are several sources of "model uncertainty" because the final failure pressure model, proposed by M. Ahammed, was obtained by mixing the equation (ii), which comes from the classical fracture mechanism theory, with several empirical equations like: eq. (vii), (to establish the relationship between hoop strength and yield strength), eq. (ix), (to calculate the folias factor) and equations (xii) and (xiii), (to calculate the corrosion growth rate).

- The sensitivity analysis demonstrated that :

  - Among the seven parameters included in the physical model, only the variability on the values of the wall thickness "T", radial corrosion growth rate "Rd" and defect depth "do", has important effect on the dispersion of the values of the remaining strength "Pf".

  - The contribution of wall thickness "T" and the defect depth "do" decreases with time, while the contribution of the radial corrosion growth rate "Rd", increases (see fig. 8). This demonstrates that "Rd" is the most important parameter to take into account in the analysis of the remaining strength in a pipeline with corrosion defects.

- Being the radial corrosion growth rate "Rd" the most critical parameter, the linear model assumed to predict the corrosion growth becomes a critical factor, regarding this issue, the following considerations are important:

  - An extensive analysis of data, coming from different services in the oil industry, Appendix #1, shows that a more accurate exponential model (also empirically gotten) can be used to estimate the radial rate of corrosion growth (Rd).

- Non empirical models, like equation (xxiv) and (xxv), are not yet in the required level of development as to be incorporated in practical applications. Nevertheless, future works in this area must be focused in the incorporation of these physically based models into the final failure pressure model for pipelines containing active corrosion defects.
- The linear model can be considered a good approximation if the period of time under consideration fall in the corrosion steady state zone.

❑ The following table resumes the results obtained using the " Advanced first order-second moment method" proposed by M. Ahammed and the results obtained with the Montecarlo approach proposed in section six, for the same set of data.

"t" (yrs)	**Advanced first order-second moment method**				**Montecarlo Simulation Method**					
	$\sigma_{Pf\ (Mpa)}$	$\mu_{Pf\ (Mpa)}$	**F_i**	R_i	$\sigma_{Pf\ (Mpa)}$	$\mu_{Pf\ (Mpa)}$	**Pf 5 % percentil**	**Pf 95 % percentil**	**F_i**	**R_i**
20	1.259	10.527	$2.238.10^{-5}$	1	1.257	10.495	8.42	12.5	$3.633.10^{-4}$	1
30	1.547	8.917	0.008	0.992	1.551	8.862	6.18	11.2	0.008	0.991
40	1.978	7.102	0.141	0.849	1.995	7.003	3.661	10.2	0.165	0.835
50	2.57	5.044	0.493	0.507	2.619	4.871	0.555	0.3	0.519	0.481

From these results, the following considerations are important:

- There is a clear "drift" in the mean of the remaining strength (Pf) and also the dispersion of its distribution increases with time (diffusion). Both trends can be detected using either one method or the other.
- The "advanced first order-second moment method" allows the estimation of the mean value and the standard deviation, but doesn't provide us the "distribution function" of "Pf". In consequence, a precise calculation of the confidence interval for this estimate is not possible with this methodology.
- The average error magnitude when using the " advanced first order-second moment method". is about 1% compared with the Montecarlo Approach.
- The "Montecarlo Approach", proposed is section six, not only provides the same benefits than the" advanced first order-second moment method", but also allows the precise estimation of

the confidence interval of “Pf”.

- Finally, it is important to highlight that, despite all the previous considerations about the limitations and benefits of the approach proposed, it can be considered as a very useful tool to optimize the decision making process about pipelines operation and maintenance; but, as most of the tools developed in this area, its successful use will depend on:
  - The accuracy and relevance of the data to be used as the inputs of the model.
  - The clear understanding of the probabilistic nature and limitations of the proposed model

  The misunderstanding of the factors previously mentioned can also lead to catastrophic consequences

8. References

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Appendix # 1 : Approach for the statistical estimation of wall thickness loss due to corrosion

Step #1: Data collection and regression analysis.

- *Data collection:*

The data was collected from “N” different points over the surface of the pipeline using ultrasonic detection, then it was registered in a data base. After a number “n” of inspections, of “N” nunber of points each, it was possible to determine the “trend” of the data over time using a plotting method, as shown in the fig. 3.2. This data corresponds to a high-pressure pipeline in the oil industry.

WALL THICKNESS DATA

		"E" = wall thickness (mm)							
No Inspection	Operational Time	Positions							
	t_i (days)	1	2	3	4	5	6	7	8
0	0	0.250	0.248	0.248	0.253	0.248	0.251	0.250	0.252
1	240	0.236	0.236	0.241	0.236	0.245	0.239	0.226	0.235
2	425	0.236	0.225	0.240	0.231	0.236	0.238	0.243	0.242
3	1139	0.215	0.223	0.223	0.215	0.212	0.221	0.221	0.215
4	1309	0.230	0.218	0.233	0.223	0.221	0.228	0.231	0.231
5	1706	0.196	0.201	0.201	0.200	0.203	0.209	0.204	0.199
6	2436	0.225	0.212	0.212	0.215	0.213	0.211	0.221	0.213
7	5968	0.216	0.202	0.201	0.188	0.204	0.199	0.208	0.210
8	6541	0.208	0.198	0.188	0.175	0.177	0.184	0.198	0.201

WALL THICKNESS VS OPERATION TIME

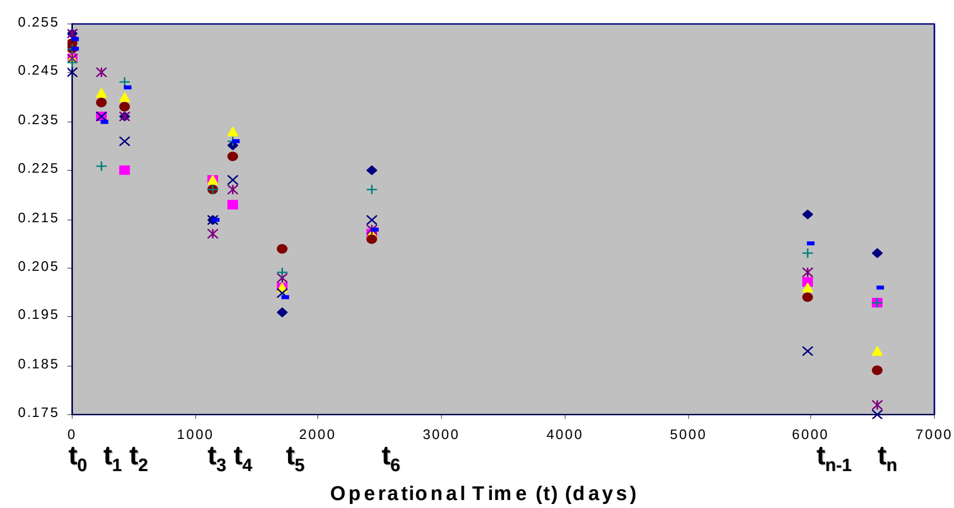
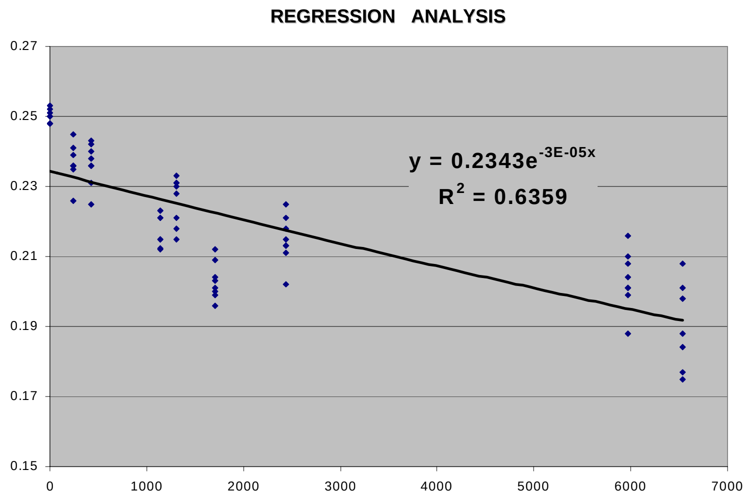


fig 3.2.
Wall thickness data from a low-pressure separation process vessel in the oil industry

- *Regression Analysis:*

This part of the methodology is focused on the development of an equation able to characterize the general behavior of the wall thickness data. This equation will be an **empirical model** of the degradation process caused by the erosion corrosion phenomenon which has been monitored for a number of years.

The general approach to fit empirical models is called regression analysis.



In this particular case, the regression analysis, shown in the above graphic, was made using Microsoft Excel 97, and the resulting empirical equation is**:**

$$E(t) = 0.2343e^{-8x10^{-6}t}$$

It is important to highlight that this empirical equation was obtained from almost 20 years of data, which corresponds to the average life service period for this kind of vessels. This fact makes possible to infer that it must have a physical meaning.

The equation can be expressed as a generic two parameters model as follow:

$$E(t) = E_0 e^{-\upsilon t} \qquad \text{(i)}$$

The analysis of this two parameters equation has led us to the conclusion that the parameter E_0 *corresponds to the initial wall thickness, and the parameter "ν" can be interpreted as a rate of wall thickness loss.*

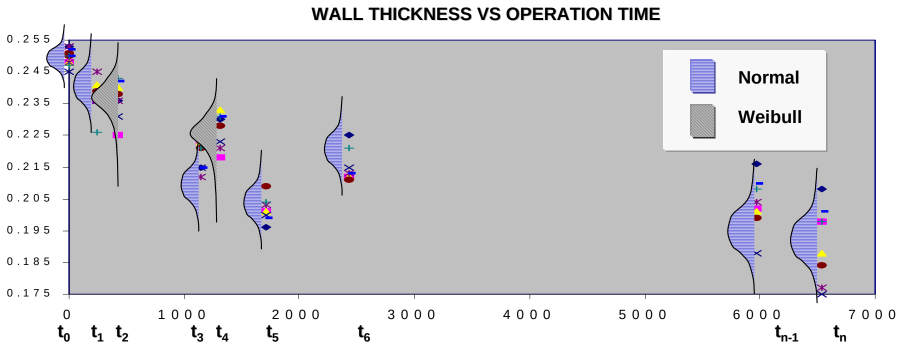
In this work, equation (i) will be used as the **empirical physical model** of the erosion corrosion process for this specific type of pipelines in this specific type of service

The graphic in the fig 3.2 shows that there is a number of values of the wall thickness "E" for each time "t_i "; these groups of values can be characterized by a probability density function of "E", (pdf(E)), for each instant of time "t_i", as it is shown in the figure 3.3 for times t_1 and t_{n-1} .

The method to find the most suitable probability distribution function of “E”, (pdf(E)) will be described in step #3.

In a similar fashion, the limit wall thickness “E_{lim}“ which has been traditionally considered as a deterministic value, can also be considered as having its own probability distribution, as the one shown in the figure 3.4.These procedure will be further explained in step #2

Step # 2: Determination of “pdf(E(t))” .



		"E" = wall thickness (mm)								Plotting results
No Inspection	Operational Time	Positions								Distribution
	t_i (days)	1	2	3	4	5	6	7	8	
0	0	0.250	0.248	0.248	0.253	0.248	0.251	0.250	0.252	Normal
1	240	0.236	0.236	0.241	0.236	0.245	0.239	0.226	0.235	Normal
2	425	0.236	0.225	0.240	0.231	0.236	0.238	0.243	0.242	Weibull
3	1139	0.215	0.223	0.223	0.215	0.212	0.221	0.221	0.215	Normal
4	1309	0.230	0.218	0.233	0.223	0.221	0.228	0.231	0.231	Weibull
5	1706	0.196	0.201	0.201	0.200	0.203	0.209	0.204	0.199	Normal
6	2436	0.225	0.212	0.212	0.215	0.213	0.211	0.221	0.213	Normal
7 = n -1	5968	0.216	0.202	0.201	0.188	0.204	0.199	0.208	0.210	Normal
8 = n	6541	0.208	0.198	0.188	0.175	0.177	0.184	0.198	0.201	Normal

fig 3.5

For each time “t_i “, (from t_1 to t_n), using plotting methods, determine the distribution that better fits over the values collected of the wall thickness “E. The graphic and the table in fig 3.5 shows that in the case under consideration, the predominant probability distribution is Gaussian or Normal. Therefore, the probability density function (pdf) can be expressed by:

$$pdf\,(E(t)) = f\,(E) = \frac{1}{\sigma_E \sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{(E(t)-\bar{E}(t))^2}{\sigma_E{}^2}\right]} \qquad \text{(ii)}$$

Combining equations (i) and (ii), the following expression can be obtained:

$$pdf\,(E(t)) = f\,(E,t) = \frac{1}{\sigma_E \sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{(E(t)-E_0 e^{-\upsilon t})^2}{\sigma_E{}^2}\right]} \qquad \text{(iii)}$$

Maximun Likelihood parameters estimation

To completely define eq. (iii), it is necessary to find the values of the parameters "ν" and "σ_E". This is possible by applying the method of Maximum Likelihood Estimator, as follows:

$$\ln(likelihood) = \Lambda = \sum_{i=1}^{n} N_i \ln\left[\frac{1}{\sigma_E E_i}\phi\left[\frac{E_i - E_0 e^{-\upsilon t}}{\sigma_E}\right]\right] \quad \text{(iv)}$$

The parameters estimate "$\hat{\upsilon}$" and "$\hat{\sigma}_E$", will be found by solving for all the evidence equation (iv), so that:

- $\frac{\partial \Lambda}{\partial \upsilon} = 0$ (v)
- $\frac{\partial \Lambda}{\partial \sigma_E} = 0$ (vi)

Once the estimators "$\hat{\upsilon}$" and "$\hat{\sigma}_E$" have been found based on the evidence, the equation (iv) is totally defined for any value of t_i, and it represents the distribution function of the stress.

$$pdf(E(t)) = f(E,t) = \frac{1}{\hat{\sigma}_E\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{(E(t)-E_0 e^{-\hat{\upsilon}t})^2}{\hat{\sigma}_E^{\,2}}\right]} \quad \text{(vii)}$$

Estimation of the pdf (E(t)) (using Mathcad 5)

Initial approximation for "thickness reduction rate": $\upsilon := 0.00001$

Initial approximation for standar deviation: $\sigma := 0.007$

Initial Wall Thicness: $Eo := 0.23$

$$\ln\left(\frac{1}{\sqrt{2\cdot\pi}\cdot\sigma}\right) - \frac{1}{2}\cdot\left(\frac{E - Eo\cdot e^{-\upsilon\cdot t}}{\sigma}\right)^2$$

ln(likelihood) function

$$\frac{(E - Eo\cdot\exp(-\upsilon\cdot t))}{\sigma^2}\cdot\exp(-\upsilon\cdot t)$$

$$\frac{-(E - Eo\cdot\exp(-\upsilon\cdot t))}{\sigma^2}\cdot Eo\cdot t\cdot\exp(-\upsilon\cdot t)$$

d(ln(likelihood))/d υ

$$\frac{-1}{\sigma} + \frac{(E - Eo\cdot\exp(-\upsilon\cdot t))^2}{\sigma^3}$$

d(ln(likelihood))/d σ

Given

$$\left[\sum_{i=0}^{n}\left[\frac{-1}{\sigma} + \frac{\left(E_i - Eo\cdot\exp\left(-\upsilon\cdot t_i\right)\right)^2}{\sigma^3}\right]\right] = 0$$

$$\left[\sum_{i=0}^{n}\left[\frac{-\left(E_i - Eo\cdot\exp\left(-\upsilon\cdot t_i\right)\right)}{\sigma^2}\cdot Eo\cdot t_i\cdot\exp\left(-\upsilon\cdot t_i\right)\right]\right] = 0$$

$$\left[\sum_{i=0}^{n}\left[\frac{\left(E_i - Eo\cdot\exp\left(-\upsilon\cdot t_i\right)\right)}{\sigma^2}\cdot\exp\left(-\upsilon\cdot t_i\right)\right]\right] = 0$$

System of Equations

$Sol := Minerr(\sigma, \upsilon, Eo)$

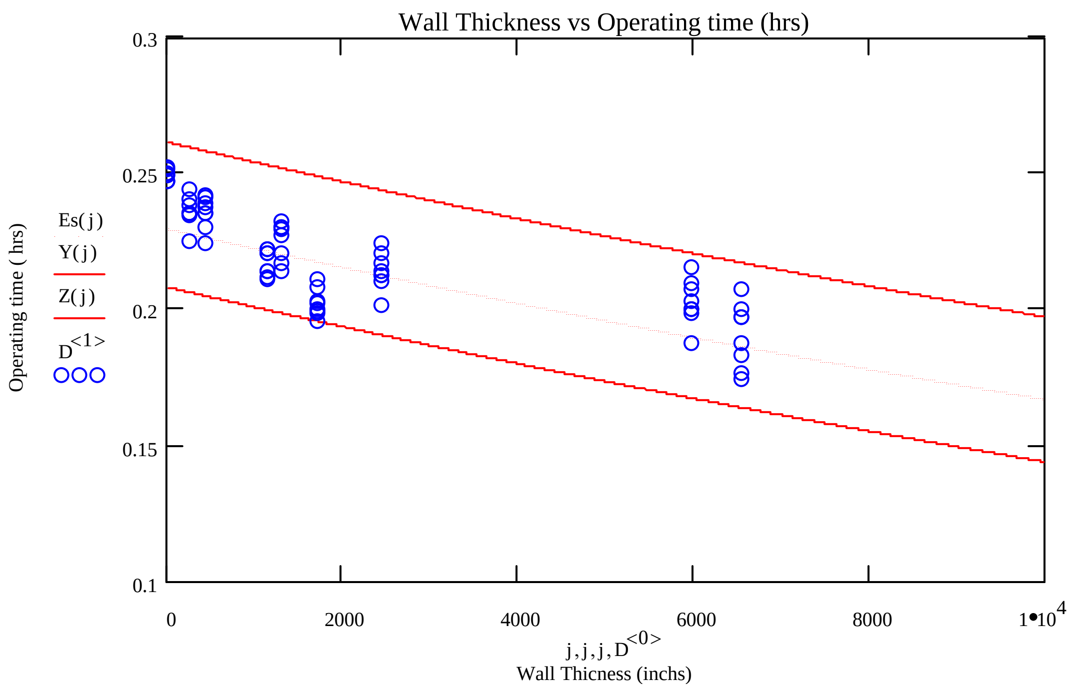
$$Sol = \begin{bmatrix} 0.011 \\ 3.195\cdot 10^{-5} \\ 0.235 \end{bmatrix}$$

Solutions for "σ**" y para "**υ**"**

$\upsilon := Sol_{1,0}$

$j := 20..\ 10000 \qquad Es(j) := Eo \cdot e^{\left(-Sol_{1,0} \cdot j\right)}$

$Y(j) := qnorm\left(0.99, Sol_{2,0} \cdot e^{-\upsilon \cdot j}, Sol_{0,0}\right) \qquad Z(j) := qnorm\left(0.01, Sol_{2,0} \cdot e^{-\upsilon \cdot j}, Sol_{0,0}\right)$



From the resuls gotten for the parameters ν and σ_E equation (vii) is now completely defined

$$pdf\,(E(t)) = f\,(E,t) = \frac{1}{0.015\sqrt{2\pi}}\,e^{-\frac{1}{2}\left[\frac{\left(E(t)-0.25e^{-4.693.10^{-5}t}\right)^2}{0.015^2}\right]} \qquad \text{(vii)}$$

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